THE HEAT AND MASS TRANSFER CHARACTERISTICS OF BOILING POINT DRYING USING RADIO FREQUENCY AND MICROWAVE ELECTROMAGNETIC FIELDS

R. M. Perkin

Electricity Council Research Centre, Capenhurst, Chester, CH1 6ES, England

(Received 21 November 1978 and in revised form 19 September 1979)

Abstract—A physical model is presented to describe the drying of unbound, pendular moisture, by a method utilising the volumetric absorption of electromagnetic energy generated at radio or microwave frequencies. The model deals with those cases where the temperature of the wet solid can be quickly raised to the boiling point of the liquid. Evaporation takes place within the pores and gives rise to an increase in pressure which drives out the vapour. Using simplified heat and mass transfer equations, the initial heating, the pressure generation and the drying of the solid are related to the properties of the wet material.

NOMENCLATURE

С,	vapour mass concentration (see Appen-
	dix 1);
Ε,	electric field strength within the wet
	material;
<i>L</i> ,	latent heat of vaporisation;
М,	moisture content (see Appendix 1);
Р,	pressure of the gaseous phase;
P _{max} ,	maximum pressure at $x = 0$;
P ₀ ,	atmospheric pressure;
<i>Q</i> ,	electromagnetic power absorbed per
	unit volume of the wet solid;
Rt,	dimensionless parameter (see equation
	12);
Τ,	temperature of the wet solid;
T_{bp} ,	normal boiling point of the liquid (see
	Appendix 1);
U _k ,	mass content of the kth phase (see
	Appendix 1);
a_{p}	permeability of the pores;
2 <i>b</i> ,	thickness of the solid;
с _р ,	effective specific heat of the wet solid;
<i>d</i> ,	average diameter of the pores;
d _e ,	effective average diameter of the pores
	for vapour flow;
j,	vapour flux per unit area at the external
$k_x, k_y,$	surface;
$n_x, n_y,$	thermal conductivity of the solid phase in the x and y directions;
r_a	specific capacity of vapour moisture;
t,	time;
t, t _{bp} , t _{max} ,	time, times to reach the temperature T_{hm} and
"bp" "max"	the pressure P_{max} ;
<i>x</i> , <i>y</i> ,	spatial co-ordinates;
γ ₀ ,	density of the dry skeleton solid;
ε",	effective loss factor of the wet material;
τ _p ,	time constant for pressure field;
<i>2</i> .	· · · · · · · · · · · · · · · · · · ·

τ _w ,	time constant for internal evaporation;
τ _h ,	time constant for initial heating of the material.

Subscripts

<i>f</i> ,	unbound liquid;
<i>s</i> ,	solid plus bound liquid;
in,	initial value.

1. INTRODUCTION

THE ABILITY of water and some solvents to absorb electromagnetic energy generated at radio and microwave frequencies forms the basis of an electrical method of moisture removal known as dielectric drying. In contrast to conventional techniques, energy is transferred directly to the liquid phase distributed throughout the volume of the wet solid. Provided the liquid can absorb sufficient electromagnetic energy, the temperature of the wet solid can be rapidly raised to the boiling point of the liquid. Vapour is evolved within the pores of the solid giving rise to an increase in pressure of the gaseous phase which drives the vapour from the material [1,2]. This drying method is particularly useful in those cases where the rate of drying by conventional means is limited by the heat and mass transfer within the wet body.

The object of this paper is to describe the physical processes involved in the dielectric drying of unbound*, pendular moisture, when the operation is carried out at the boiling point temperature of the liquid. A simplified model of the wet material is used to formulate appropriate heat and mass transfer equations. The solutions to these equations enable the drying process to be related to the physical properties of the wet solid and provide certain criteria for the use of this drying method. First of all, the principles underlying the absorption of electromagnetic energy are outlined so that the way in which the liquid and solid phases determine the energy absorption can be appreciated.

^{*} See Appendix 1 for a glossary of terms.

2. VOLUMETRIC ABSORPTION OF ELECTROMAGNETIC ENERGY

The volumetric absorption of electromagnetic energy by a wet solid is due primarily to the response of the unbound liquid to the applied electric field. Water and some solvents are able to absorb energy because of their polar structure and the presence of salts dissolved in the liquid.

The absorption and dissipation of electromagnetic energy as heat in unbound polar liquids can be interpreted either in terms of the breaking of chemical bonds between adjacent molecules, or alternatively, as the internal friction between molecules as they rotate in an attempt to realign their electric dipole moments with the changing electric field [3]. Both models give equivalent results. The amount of energy dissipated is directly proportional to the loss factor of the liquid ε'_{D} a dimensionless, frequency dependent parameter determined by the molecular structure of the liquid [4]. Normally at radio frequencies the motion of the polar molecules results in relatively little energy absorption whereas at microwave frequencies significant amounts of energy can be absorbed since the maximum value of the loss factor ε''_D lies in this frequency range.

If dissolved salts are present in the unbound liquid, electromagnetic energy can be dissipated as heat through the motion of the ions in the electric field. This form of heating depends on the electrical conductivity of the liquid σ and is approximately frequency independent: it contributes equally in both the radio and microwave frequency ranges [4]. These two heating mechanisms for a liquid can be formally represented in terms of an effective loss factor given by

$$\varepsilon_f'' = \varepsilon_D'' + \text{constant} \cdot \sigma/f$$
 (1)

where f is the frequency of the applied field. Both $\varepsilon_D^{"}$ and σ are temperature dependent: $\varepsilon_D^{"}$ decreases and σ increases with increasing temperature.

The effective loss factor of a wet body ε'' is derived from that of the dry skeleton solid, the bound and unbound liquid, and can be found by several mixture theories [5,6]. Provided the unbound liquid is present in sufficient quantity it largely determines the value of ε'' . Measured data are usually presented as plots of loss factor ε'' against the average moisture content of the wet body at a given temperature, as Fig. 1 illustrates for white wool at room temperature and a frequency of 27 M Hz [7]. It can be seen that the contribution from the solid phase, $\overline{M} = 0$, is small. When liquid is first added to the dry material it is bound to the solid phase and its movement is inhibited. Consequently this bound liquid is only capable of absorbing a small amount of energy, region AB. As further layers of liquid are added to the bound layer they increasingly exhibit the properties of the bulk or unbound liquid and the loss factor increases rapidly in proportion to the quantity of liquid added, region BC.

The power absorbed per unit volume of the wet solid is given by

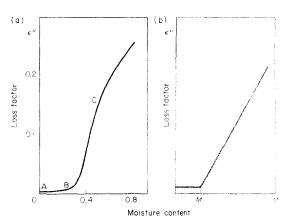


FIG. 1. Variation of loss factor with moisture content: (a) ε'' versus the average moisture content for white wool, (b) simplified variation used in the model.

$$Q = 2\pi f \varepsilon_0 \varepsilon'' E^2 \tag{2}$$

where ε_0 is the permittivity of free space, *f* the frequency of the applied electromagnetic field and *E* is the amplitude of the electric field within the solid. The magnitude and direction of *E* depends on the orientation of the external electric field relative to the surface of the solid and the value of dielectric constant of the wet solid. To avoid the problems of arcing and electrical breakdown the magnitude of the electric fields associated with dielectric dryers must not exceed about 200 kV/m. In many forms of dielectric dryer the external electric field is approximately parallel to the surface of the material and the internal field can be taken as constant; such a case is assumed in the present model.

3. DESCRIPTION OF THE DRYING PROCESS

Before introducing the specific model used in the paper, a general view of the drying will be outlined. The sequence of events described below is confined to those operations where the pendular liquid can be rapidly heated to its boiling point temperature and most of the moisture removal occurs once this temperature has been attained. In many practical situations either the wet solid is preheated before the dielectric drying stage or the required moisture removal is relatively large so that the above simplification applies. When the electromagnetic field is applied to the wet material the temperatures of the liquid and solid phases generally increase at different rates and heat conduction occurs between the two phases. At the interface between the liquid and gaseous phases local thermodynamic equilibrium exists so that the partial pressure of the vapour can be related to the temperature of the unbound liquid through the Clausius Clapeyron equation.

Part of the energy absorbed by the liquid is used in evaporation. The vapour evolved within the pores lead to an increase in the pressure P, the sum of the partial pressures of the vapour and air, above the atmospheric pressure P_0 . Two mass transfer processes take place: molecular diffusion of the air and vapour superimposed on a convective flow of the whole gaseous phase which is driven by the difference between the pressure in the pores and the atmospheric pressure at the surface of the solid. The convective velocity v_c is related to the pressure gradient by $v_c \propto a_p \cdot \nabla P$, where the permeability a_p is a function of the geometry and size distribution of the pores [8]. While the temperature of the liquid is well below its normal boiling point T_{bp} diffusion is the dominant flow mechanism. Near the boiling point the situation is reversed and when the liquid reaches the temperature T_{bp} the air is expelled from the pores which are filled with vapour; the mass transfer is now due solely to the convective flow.

With further internal evaporation, the pressure of the vapour rises above atmospheric pressure and causes an elevation in temperature of the liquid which can be determined from the Clausius Clapeyron equation. This means that unlike an open liquid surface evaporating into the atmosphere, not all of the absorbed energy can be used for evaporation since some of it is consumed in raising the temperature of the liquid, solid, and gaseous phases. Without the need to raise the temperature of the wet material, the rate of pressure rise would be extremely rapid. The effect of the temperature rise is to reduce the rate of pressure increase which can, however, still be relatively quick compared to the rate of internal evaporation. As liquid evaporates the loss factor ε'' decreases and, with the electric field strength remaining approximately constant, causes a reduction in the absorbed power. The result is that the pressure in the pores quickly reaches a maximum value. The majority of the moisture removal occurs after the maximum pressure has been attained. All the energy absorbed by the unbound liquid is now used for the internal evaporation. Since the absorbed power Q is proportional to the amount of unbound liquid present, the rate of evaporation decreases as the solid dries out. In addition the permeability a_p increases so that the resistance to vapour flow, which is proportional to $1/a_p$, diminishes. The outcome of these changes is for the pressure and rate of moisture removal to slowly decline.

As the power absorption due to the unbound liquid decreases so the power absorbed by the solid and bound liquid, although small in absolute value, can influence the drying. While the pockets of unbound liquid act as an effective sink for the energy absorbed by the solid phase, its temperature will not increase since the energy is used for evaporation of the liquid. However, if isolated dried-out regions exist, then in these parts the energy absorbed by the solid raises the local temperature of the solid phase which can lead to overheating of the material. In many practical situations it appears that the liquid sinks remain effective down to the bound liquid level, e.g. [9]. This situation arises automatically in the present model as a consequence of the assumption that the thermal conductivity k_{y} is infinite (see Section 4.2).

Once the unbound and loosely bound moisture have

been evaporated and removed leaving only tightly bound liquid, it usually becomes difficult to efficiently transfer the electromagnetic energy into the solid owing to the small value of the loss factor. Although bound moisture can be driven off, the most significant effect is likely to be the heating of the solid phase as discussed above.

The flow of vapour from the wet material into the surrounding air stream is analogous to the transpiration cooling of a surface by gas injection into the boundary layer [10]. At the boiling point T_{bp} the mass flow away from the surface is determined by the pressure generated within the material and is independent of the flow of the external air stream. Detailed modelling of the external flow, therefore, is not required. The boundary conditions for the pressure equation enable the flux away from the surface to be determined. The role of the air stream is to prevent heat losses from the wet solid and to this end the temperature of the air stream and the surrounding enclosure should be set to a value greater than the maximum value reached by the wet material.

4. DEVELOPMENT OF A PHYSICAL MODEL AND THE REPRESENTATION OF THE WET MATERIAL

The rigorous derivation of heat and mass transport equations which are applicable to drying operations has only recently been attempted [11]. Most theoretical treatments have used an intuitive approach which ignored the discrete nature of the different phases within the solid and instead assumed that the wet material could be treated as a 'continuum' described by the familiar transport equations [12, 13].

In the present case the aim is to develop a picture of dielectric drying under certain conditions and relate it to the physical properties of the wet solid. For this reason it is considered sufficient to use a simplified, hypothetical model of the wet solid which retains the essential characteristics yet allows the limitations and restrictions which must be made in the mathematical modelling to be clearly seen.

4.1. Pore structure

The solid is taken to be an infinite slab of thickness

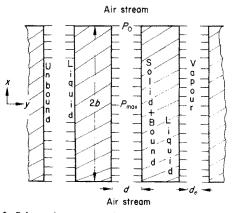


Fig. 2. Schematic representation of the idealised solid used in the model.

2b, see Fig. 2. Both surfaces are exposed to an external air stream so that the drying is symmetric. Only materials with pore diameters greater than 10^{-7} m, that is, essentially non-hygroscopic bodies, will be considered. The small proportion of bound liquid which does exist is confined to a thin layer in contact with the pore walls. Those pores which reach the surface of the solid are represented by a parallel array of cylindrical capillaries of length 2b and diameter d, where d is the average pore size as determined from the actual pore distribution. By assuming that the vapour flow through the pores can be represented as 'Hagen-Poiseuille' type flow through a tube it is found that $a_p \propto d_e^2 / \Delta \sim d^2 / \Delta$ where d_e is the effective average diameter for vapour flow and Δ is a tortuosity factor which accounts for the true length of the pores. As the moisture evaporates d_e and hence a_p increases.

4.2. Thermal properties and heat transfer

Strictly speaking, the heat conduction in the solid and gaseous phases requires a two dimensional model. However, in order to make the mathematical analysis practicable, it is assumed that the gas phase has the same temperature as the unbound liquid and that the thermal conductivity, k_y , of the solid is infinite. The model becomes one dimensional with changes only occurring in the x direction: at a given level x the temperature of the three phases are equal. An effective specific heat for the wet solid c_p can then be defined by

$$\gamma_0 c_p = \sum_k c_{pk} U_k$$

where c_{pk} and U_k are the specific heat and mass concentration of the kth phase.

An alternative to the above simplification is the other extreme where the thermal conductivity k_y is taken as zero. In this case the heating of the solid phase becomes completely independent of the processes occurring in the liquid and gaseous phases and can represent, as a first approximation, the situation when solids of very low thermal conductivity are dried. The solutions when $k_y = 0$ for the initial temperature rise of the liquid, and the pressure evolution, can be found from those for $k_y = \infty$ by omitting the contributions from the solid phase in the absorbed power and effective specific heat terms. The above two extremes for the thermal conductivity k_y yield the limiting drying characteristics between which the real drying behaviour will lie.

At the boundary between the solid and the external air stream the heat flux is assumed to be negligible and for simplicity $k_x(\partial T/\partial x)$ at $x = \pm b$ is equated to zero. The heat fluxes due to the internal convective flow of the vapour and the conduction in the solid and gaseous phases in the x direction are neglected. In the first case it can be shown that the convective heat flux is negligible in comparison with the volumetric energy absorption term while the latter case is a result of a further assumption that the pressure field is established before significant heat conduction in the x direction takes place.

4.3. Absorption of electromagnetic energy

The variation of loss factor with moisture content is assumed to be that shown in Fig. 1(b): the knee in the curve at which the proportion of unbound liquid increases rapidly is replaced by a sharp transition at a moisture content M_c . The location of the point M_c is not uniquely defined since it depends on how the simplified loss curve is fitted to the experimental data. The exact shape of this simplified loss factor curve is, however, unimportant in the drying model. The contribution to the loss factor from the solid phase and bound liquid is assumed to be constant while that from the unbound liquid varies linearly with moisture content and temperature. For a given frequency the absorbed power Q is then given by

$$Q = Q_s + Q_f \tag{3}$$

where Q_s and Q_f represent the power absorbed per unit volume of wet solid by the solid plus bound liquid, and the unbound liquid respectively. With the electric field strength constant, the power Q_f follows the changes in the loss factor with temperature and moisture content:

$$Q_f \propto (M - M_c) (T - T_m). \tag{4}$$

4.4. Moisture distribution

In a real solid the isolated sites containing liquid can be fed by local capillaries but on a macroscopic scale movement of the liquid phase is not possible. Despite the discrete distribution of the liquid most theories assume that localised values can be assigned to its mass concentration and related to a continuum transport equation [13]. To represent these characteristics a hypothetical model for the liquid phase is assumed: unbound moisture is distributed continuously on the pore walls without actually filling the pores, see Fig. 2. Liquid flow is prohibited and the thermal conductivity of the water in the x direction is taken as zero. The small amount of bound moisture is considered to be part of the solid skeleton. At the interface between the phases local thermodynamic equilibrium is assumed to exist.

4.5. Division of the drying operation

In line with the physical picture presented, it is expedient to assume that during the period when the temperature of the liquid rises to the boiling point T_{bp} evaporation of moisture is negligible. On this basis the pressure within the pores does not increase above atmospheric pressure until this temperature is attained. For simplicity the initial moisture content and temperature distributions are assumed to be uniform. The drying process can then be divided into three periods:

- (1) the initial heating of the solid to the normal boiling point temperature T_{bp} ,
- (2) the generation of the pressure field and the attainment of a maximum pressure,
- (3) the moisture removal and resulting decline in the pressure as the absorbed power decreases.

Table 1. The limitations of the model and the simplifications employed in solving the transport equations

Limitations

- 1. Non-hygroscopic solid; pore size $> 10^{-7}$ m.
- 2. $P \leq 1.5P_0$: linearised pressure equation.
- 3. Drying conducted at boiling point T_{bp} . 4. Pressure field set up rapidly: $\frac{Rt\tau_p}{r} < 1$.
- 4. Pressure field set up rapidly: τ_w

Simplifying assumptions

- 1. Idealised pore model.
- 2. Three stage model: (i) heating, (ii) pressure generation, and (iii) moisture removal.

2(a). During (i)
$$\frac{\partial M}{\partial t} = \frac{\partial P}{\partial t} = 0.$$

- 3. Thermal conductivity $k_{y} = \infty$: gives one dimensional model.
- 4. No heat flux across surface $k_x \frac{\partial T}{\partial x} = 0$ at $x = \pm b$.
- 5. The permeability a_p is constant
- 6. Pressure field set up before significant heat conduction in the solid phase takes place-simplifies equations.
- 7. Q_f varies linearly with \hat{T} and M.
- 8. Elevation of temperature above T_{bp} is small enough to assume that $\partial T/\partial P$ is constant, and

$$Q_f = Q_{bp} \left(\frac{M - M_c}{M_{in} - M_c} \right)$$

9. Time variation of M follows the solution found when Rt= 0.

Simplifying conditions

- 1. The electric field E is constant.
- 2. The initial moisture and temperature profiles are uniform.
- 3. Symmetric drying $\frac{\partial P}{\partial x} = 0$ at x = 0.

5. TRANSPORT EQUATIONS AND THEIR SOLUTION

Transport equations can be derived in a straightforward manner for the idealised model using the shell balance method [13, 14]. The assumptions and limitations used in the model and the further restrictions employed in solving the equations are listed for convenience in Table 1.

5.1. Initial heat period

With insignificant evaporation and no heat transfer across the external surface of the solid the equation describing the rate of increase in temperature of the wet body reduces to

$$\gamma_0 c_p \frac{\partial T}{\partial t} = Q_f = Q_{in} + (Q_{bp} - Q_{in}) \frac{(T - T_{in})}{T_{bp} - T_{in}} \quad (5)$$

where for simplicity the contribution to the absorbed power from the solid and bound liquid Q_s has been neglected, and Q_{bp} and Q_{in} represent the power absorbed per unit volume of wet solid by the unbound liquid at the initial moisture M_{in} and the temperatures T_{bp} , T_{in} respectively. Solving this equation gives

$$\frac{T-T_{in}}{T_{bp}-T_{in}} = \frac{\alpha}{1-\alpha} \left[\exp \frac{(1-\alpha)t}{\tau_h} - 1 \right]$$
(6)

where $\alpha = Q_{in}/Q_{bp}$ and the time constant $\tau_h = \gamma_0 c_p (T_{bp})$ $T_{in})/Q_{bp}$ represents the time which would be required to heat the wet body to the boiling point T_{bp} if the absorbed power were constant at the value Q_{bp} . For dipolar loss at microwave frequencies the value of α is about five while for conductive heating at radio frequencies $\alpha \leq 1$. The time to reach the temperatures T_{bn} is given by

$$t_{bp} = \frac{\tau_h}{1 - \alpha} \log_e \frac{1}{\alpha}.$$
 (7)

5.2. Generation of the pressure field

The continuity equation for the pressure P, or equivalently the vapour concentration C, is given by

$$\frac{\partial P}{\partial t} = a_p \frac{\partial^2 P}{\partial x^2} - \frac{1}{r_a} \frac{\partial M}{\partial t},$$
(8)

with the pressure and vapour concentration related by

$$C = \gamma_0 r_a P. \tag{9}$$

The equation shows that the rate of pressure increase within an element volume is determined by the rate at which vapour is evolved by internal evaporation, and by the net flow of vapour out of the volume. In deriving this equation terms in P have been linearized so that the equation is only accurate for values of the pressure less than about $1.5 P_0$. This limitation is not serious since many drying materials cannot sustain pressures greater than this. The solutions to the pressure equation can still be used to predict the occurrence of large pressures within the pores even though the magnitudes will be wrong. The permeability a_p has been taken as constant; as will be seen its actual variation with moisture content is unimportant in the model. At the surface of the solid the pressure is equal to the atmospheric pressure and as the drying is symmetric, the pressure gradient at x = 0 is zero. The initial boundary conditions corresponding to the instant when the temperature of the solid reaches T_{bp} is given by $P(x) = P_0$. The corresponding equation for the temperature changes is given by

$$\gamma_0 c_p \frac{\partial T}{\partial t} = \gamma_0 L \frac{\partial M}{\partial t} + Q_f. \tag{10}$$

Since $Q_f > Q_s$, the power absorbed by the solid has again been neglected for simplicity.

Formally the characteristics of the dielectric drying are exhibited by the solution to the pressure equation which is obtained when the temperature elevation is neglected, that is when $\partial T/\partial t$ is set equal to zero.

The significant terms are a relatively slow temporal variation proportional to exp $-(t - t_{bp})/\tau_w$ and a much faster variation proportional to

$$\exp -\frac{\pi^2}{4} \frac{(t-t_{bp})}{\tau_p}$$

where

$$\tau_w = \frac{\gamma_0 (M_{in} - M_c) L}{Q_{bp}} \quad \text{and} \ \tau_p = \frac{b^2}{a_p}.$$

The time constant τ_w represents the time it would take to evaporate the unbound moisture if the power were maintained at the value Q_{bp} ; τ_p is the time constant to establish the pressure field. The fast variation describes the initial build-up of the pressure while the slower term is related to the moisture removal in the third period.

The maximum pressure P_{max} which is generated at the centre of the material is given by

$$P_{max} - P_0 = \frac{1}{2r_a \gamma_0} \left(\frac{Q_{bp} b}{L}\right) \left(\frac{b}{a_p}\right) = \frac{M_{in} - M_c}{2r_a} \left(\frac{\tau_p}{\tau_w}\right).$$
(11)

In the time taken to achieve this pressure P_{max} the actual change in a_p would be small so that the simplification that a_p is constant is justified. For $P_{max} - P_0 \leq 1.5 P_0$ it can be shown from the above expression that typically $\tau_w/\tau_p \gtrsim 100$.

The non-dimensionalized pressure at the centre of the solid,

$$p = \frac{2r_a}{M_{in} - M_c} \left(\frac{\tau_w}{\tau_p}\right) (P - P_0).$$

is plotted against the time $(t - t_{bp})/\tau_w$ in Fig. 3 for various values of the ratio of the time constants τ_w/τ_p . It can be seen that for $\tau_w/\tau_p > 1$ the duration of stage 2 is small in comparison with that of stage 3. The curves for $\tau_w/\tau_p \leq 1$ have been included to show that for these

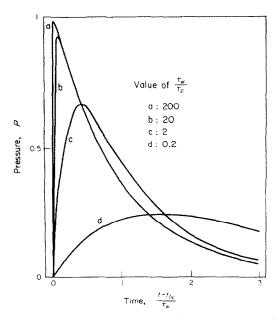


Fig. 3. Plot of the non-dimensionalised pressure p at x = 0against the time $(t - t_{bp})/\tau_w$, when Rt = 0, for various values of the ratio τ_w/τ_p .

smaller values of τ_w/τ_p the actual pressures generated $(\propto \tau_p/\tau_w)$ would be large: the linear theory would be inapplicable in such cases.

The influence of the temperature elevation due to the increase in pressure can be characterised by a dimensionless number

$$Rt = -\frac{c_p}{Lr_a} \frac{\mathrm{d}T}{\mathrm{d}P}.$$
 (12)

The smaller the value of Rt the less the temperature rise affects the drying characteristics; neglecting the effect corresponds to putting Rt = 0. In the restricted pressure range considered here, $P \leq 1.5 P_0$, the values of Rt and τ_w/τ_p are such that $\tau_w/\tau_p \gg 1$ and $Rt \cdot \tau_p/\tau_w < 1$. In these circumstances the solution to the equations taking into account the temperature rise can be obtained by a perturbation technique from the solution for Rt = 0: a factor 1/(1 + Rt) is included in the previous fast exponential term to give

$$P - P_{0} = \frac{Q_{bp}b^{2}}{2r_{a}\gamma_{0}La_{p}} \left[\left(1 - \frac{x^{2}}{b^{2}} \right) \exp - \frac{(t - t_{bp})}{\tau_{w}} - \frac{32}{\pi^{3}} \sum_{n=0}^{\infty} \frac{(-1)^{n}}{(2n + 1)^{3}} \cos \frac{(2n + 1)\pi x}{2b} + \exp - \frac{(2n + 1)^{2}\pi^{2}(t - t_{bp})}{4\tau_{p}(1 + Rt)} \right].$$
 (13)

In arriving at the above expression for the pressure, both dT/dP and c_n have been taken as constant, the latter at its initial value corresponding to $M = M_{0}$. The term for the energy absorbed by the unbound liquid Q_f has been linearised by setting it equal to Q_{by} $(M - M_c)/(M_{in} - M_c)$: the change of Q_f with temperature above T_{bp} has been ignored. Furthermore in the expression for Q_{f} the temporal variation of the moisture content M is taken to be that found from the solution for Rt = 0. The first two simplifications are justified as long as the elevation in temperature above T_{bp} and the change in Q_f are small. In any case, order of magnitude estimates of the pressure increase will not be seriously affected unless Q_f changes dramatically with temperature which is unlikely. The approximation for M is applicable provided $Rt \cdot \tau_p/\tau_w < 1$.

The significance of the (1 + Rt) factor is that it slows down the rate of pressure increase in the early stage in accordance with the physical model presented. The maximum pressure P_{max} is still of the same magnitude as for Rt = 0 and is achieved in the time interval

$$t_{max} - t_{bp} = \frac{4(1+Rt)}{\pi^2} \tau_p \log_e \frac{8\tau_w}{\pi(1+Rt)\tau_p}.$$
 (14)

which remains small compared to the duration of stage 3 provided $Rt\tau_p < \tau_w$.

5.3. Moisture removal

The moisture removal which occurs after the maximum pressure has been reached is determined by the vapour flux per unit area at the surface given by

$$j = -\gamma_0 r_a a_p \frac{\partial P}{\partial x_{x=\pm b}}.$$
 (15)

In this period the time variation of the pressure is relatively slow and the approximation $\partial P/\partial t = 0$ can be made in the transport equations. The heat transfer equation then simplifies to

$$\frac{\partial M}{\partial t} = -\frac{Q_{bp}}{L\gamma_0} \frac{M - M_c}{M_{in} - M_c},$$
(16)

which leads to the following expression for the vapour flux at the surface

$$j = \frac{Q_{bp}b}{L} \exp -\left(\frac{t - t_{bp}}{\tau_w}\right). \tag{17}$$

If the final desired moisture content is close to M_c then the contribution from the energy absorbed by the solid and bound liquid Q_s must be included to give

$$j = \frac{(Q_{bp} + Q_s)b}{L} \exp - \left(\frac{t - t_{bp}}{\tau_w}\right).$$
(18)

Without the energy Q_s the time required to reduce the moisture content to M_c would be infinite whereas with Q_s included the time remains finite.

6. DISCUSSION

The expressions derived from the simplified model show that the drying process is determined principally by the time constants τ_h , τ_w and $\tau_p/(1 + Rt)$. These in turn are related to the power absorption per unit volume Q, the thickness of the solid 2b, the resistance to vapour flow b/a_p which in turn depends on the pore size since $a_p \propto d^2$, and the effective specific heat c_p . The evolution of the pressure field and the magnitude of the pressure generated are readily understood in terms of these basic parameters.

6.1. Generation of the pressure field

As already seen, the pressure reaches a maximum value and then declines along with the absorbed power as the solid dries out. Assuming for the moment that all the absorbed energy is used for the evaporation, Rt = 0, the time t_{max} to reach the maximum pressure is determined by the rate at which vapour is generated to produce the increase in pressure $(\propto 1/\tau_w)$, and the time constant τ_p for the establishment of the pressure field. If the resistance to vapour flow b/a_p is small so that $\tau_p < \tau_w$ the driving force $\partial P/\partial x$ is rapidly set up enabling the vapour flow to match the rate of internal evaporation. Under these conditions the time t_{max} is relatively insensitive to τ_w and $t_{max} \propto \tau_p$.

When the temperature elevation is included, only an amount of power of the order of $Q_f/(1 + Rt)$ is initially available for evaporation, increasing to Q_f as the rate of temperature rise $(\propto \partial P/\partial t)$ goes to zero. Vapour is generated more slowly as compared to the Rt = 0 case and the pressure gradient is set up (1 + Rt) times slower so that $t_{max} \propto \tau_p (1 + Rt)$.

As there is relatively little resistance to the vapour flow when $\tau_p < \tau_w$ the pressure field can quickly respond to the slow changes in the internal evaporation rate. In this way during the third stage of drying, the pressure and vapour flow follow the time dependence of the power absorption. When $\tau_p > \tau_w$ the vapour flow, owing to the large resistance, cannot match the rate of vapour generation until a large pressure gradient is produced. Consequently larger pressures, as compared to the case $\tau_p < \tau_w$, and longer times t_{max} , relative to the evaporation time τ_w , are to be expected. In the third stage the large resistance to the flow prevents the pressure field from following the changing rate of evaporation.

The maximum pressure is attained when $\partial P/\partial t = 0$ and evaporation is due to the absorbed power $Q_f \sim Q_{bp}$. The total rate of internal evaporation per unit area of the solid is given by bQ_{bp}/L and since the pressure does not increase this is matched by the flow of vapour at the surface which is found from equation (15) and is of the order of

$$r_a \gamma_0 a_p \frac{(P_{max} - P_0)}{b}.$$

Equating the two expressions gives

$$P_{max} - P_0 \sim \frac{1}{r_a \gamma_0} \frac{Q_{bp} b}{L} \frac{b}{a_p}$$

in general agreement with the more exact form, equation (11). As the resistance to flow b/a_p and the rate of internal evaporation decrease, so the maximum pressure produced also decreases.

6.2. Equivalent drying model

It has been seen that when $\tau_w > \tau_p Rt$ the maximum pressure is attained in a short time compared to the subsequent period of moisture removal and the equations describing this third period can be simplified using $\partial P/\partial t = 0$. The expression for the vapour flux at the surface, equation (17), depends on the power absorption but not on the pore structure of the solid as represented through the permeability a_p . In these circumstances the structure of the solid, from the point of view of moisture removal, is unimportant and a drying model using a 'free' surface covered with an equivalent amount of liquid can be used.

6.3. Levelling of moisture profiles

The simplified equations for the third period can be used to bring out an important feature of dielectric drying, namely the ability to reduce any nonuniformities in the moisture distribution in a wet body [9, 15]. Equation (16) shows that the rate of internal evaporation at a point is directly proportional to the amount of liquid present. Consequently with an initial non-uniform moisture profile the fastest rate of evaporation occurs where the solid is wettest. As the drying proceeds so the gradient in the moisture profile 'levels' out.

In the present case this characteristic has been obtained because the electric field was taken as constant and the loss factor ε'' varied linearly with moisture content. In general, provided the overall result of any changes in the loss factor and electric field is for the absorbed power Q to decrease with decreasing moisture content, this levelling effect will be obtained.

6.4. Pressure limitations and power input

The second period which describes the pressure build up is very important since it relates the maximum pressure generated to the absorbed power density. Rearranging the expression for the maximum pressure gives

$$Q_{bp} = 2r_a \gamma_0 L (P_{max} - P_0) \frac{a_p}{h^2},$$
 (19)

which allows estimates of the maximum permissible value of the power Q_{bp} to be found. The maximum tolerable power is directly proportional to the square of the pore diameter, since $a_p \propto d^2$, and inversely proportional to the square of the thickness of the solid. For a given value of the pressure P_{max} the permissible value of Q_{bp} for thick, fine pored materials is less than that for thin, coarse pored bodies.

It must be remembered that the permeability a_p is determined from the average pore size. When the variation in pore sizes is large, it is possible for the pressure in the fine pores to exceed the permissible value while the pressure in the coarse pores is below this value. The pore size distribution will determine the tolerance which must be placed on a_p when estimating the maximum permissible power absorbed Q_{bp} . Whether this maximum value of the absorbed power can be achieved, however, depends on the loss factor ε'' and the maximum electric field strength which can be used before breakdown problems occur. In circumstances where the loss factor is relatively small, the maximum allowable electric field rather than the maximum pressure will limit the value of the absorbed power.

7. CONCLUSIONS

The simplified drying model considered materials which are essentially non-hygroscopic apart from a small proportion of bound liquid. In such cases the main sink for the electromagnetic energy is initially provided by the unbound liquid. If the required final moisture content approaches the bound moisture limit however, it is necessary to consider the energy absorbed by the solid and bound liquid.

The heat and mass transfer model developed for the dielectric drying divides the drying into three phases: initial heating of the wet material to the boiling point T_{bp} , generation of a pressure field, and the internal evaporation and subsequent removal of moisture. In the practical cases covered by the model the duration of the second period is small.

The solution of the simplified transport equations showed that the drying process is related to several time constants τ_h , τ_w , $\tau_p/(1 + Rt)$ which are determined principally by the power absorbed by the wet solid, the pore structure of the solid and the effective specific heat of the wet body. When the time constant for the pressure field τ_p is smaller than the evaporative time constant τ_w the moisture removal is determined solely by the effective power available for evaporating the moisture: the pore structure does not seriously impede the vapor flux. Provided the absorbed power per unit volume of the wet solid decreases as the moisture content decreases, then dielectric drying will lead to the smoothing out of non-uniform moisture distributions.

In general the absorbed power densities must not exceed a critical value determined either by the maximum excess pressure which can be tolerated within the solid or by the maximum electric field which can be applied. In order to calculate the drying and pressure curves the dielectric properties and permeability of the solid must be known.

Acknowledgement— This paper is published by permission of the Director, the Electricity Council Research Centre.

REFERENCES

- V. T. Mustyatsa and E. V. Verlon, Periodic dielectric heating of wet materials, *Electro Chem. Ltd. Ind. Proc. and Biology* No. 4, 61-64 (1975).
- D. W. Lyons, J. D. Hatcher and J. E. Sunderland, Drying of a porous medium with internal heat generation, *Int. J. Heat Mass Transfer* 15, 897–905 (1972).
- 3. J. B. Hasted, Aqueous Dielectrics. Chapman and Hall (1973).
- S. S. Stuchly and M. A. K. Hamid, Physical parameters in microwave heating processes, J. Micro. Power 7, 117–137 (1972).
- A. Kraszewski, Prediction of the dielectric properties of two phase mixtures, J. Micro. Power 12(3), 215-222 (1977).
- W. R. Tinga, W. A. Voss and D. F. Blossey, Generalized approach to multiphase dielectric mixture theory, J. Appl. Phys. 44, 3897-3902 (1973).
- 7. J. L. Driscoll, Measurements on the Dielectric Properties of Wool and other Textile Fibres at Radio Frequencies, ECRC Memorandum M659 (1973). (A copy is held by the British Lending Library, Boston Spa, Yorkshire.)
- F. A. L. Dullien, New network permeability model of porous media, A.I.Ch.E. Jl 21(2), 299-307 (1975).
- 9. R. Morrow, Moisture levelling in the radio frequency drying of green beech timber. Proceedings of the Second Australasian Conference on Heat and Mass Transfer, pp. 437-444. (February 1977.)
- 10. E. R. G. Eckert and R. M. Drake, *Heat and Mass Transfer*, 2nd edition. McGraw-Hill (1959).
- S. Whitaker, Simultaneous heat, mass and momentum transfer in porous media. A theory of drying, Adv. Heat Transf. 13, 119-203 (1977).
- 12. A. V. Luikov, Heat and Mass Transfer in Capillary Porous Bodies. Pergamon Press, Oxford (1966).
- T. Z. Harmathy, Simultaneous moisture and heat transfer in porous systems with particular reference to drying. *I/EC Fundamentals* 8(1), 92-103 (1969).
- 14. R. B. Bird, W. E. Stewart and E. N. Lightfoot, Transport Phenomena. John Wiley (1960).
- P. L. Jones, J. Lawton and I. M. Parker, High frequency paper drying—I. Paper drying in radio and microwave frequency fields, *Trans. Instn. Chem. Engrs* 52(2), 121-131 (1974).

APPENDIX 1: GLOSSARY OF TERMS

Bound liquid

A liquid whose properties are modified due to the interaction with a solid phase through chemical or Van der Waal forces.

Loss factor

A dimensionless parameter related to the electrical properties of the medium; it determines the possible energy absorption.

Mass concentration of a phase

Mass of a given phase held in a unit volume of the wet material.

Microwave frequencies

915 MHz, 2450 MHz are commonly used for dielectric heating.

Moisture content

The mass concentration of the liquid phase divided by the mass concentration of the solid phase, i.e. the density of the skeleton solid.

Normal boiling point

The temperature at which the saturated vapour pressure of the liquid is equal to atmospheric pressure.

Pendular moisture

Liquid held in isolated pockets within the pores of the solid.

Radio frequencies

13.5 MHz, 27.12 MHz are used for dielectric heating.

Unbound liquid

A liquid which exhibits the properties of the bulk fluid and exerts its full vapour pressure.

LES CARACTERISTIQUES THERMIQUES ET MASSIQUES DU POINT D'EBULLITION EN SECHAGE A PARTIR DES FREQUENCES RADIO ET DES CHAMPS ELECTROMAGNETIQUES DE FREQUENCE MICRO-ONDE

Résumé—On présente un modèle physique pour décrire l'humidité pendulaire par une méthode utilisant l'absorption volumétrique de l'énergie électromagnétique aux fréquences radio ou micro-ondes. Le modèle concerne les cas où la température du solide humide peut être rapidement élevée jusqu'au point d'ébullition du liquide. L'évaporation apparait dans les pores et provoque l'accroissement de pression qui chasse la vapeur. En utilisant des équations simplifiées pour le transfert de chaleur et de masse, le chauffage initial, la pression et le séchage du solide sont reliés aux propriétés du matériau humide.

DER WÄRME- UND STOFFÜBERGANG BEIM VERDAMPFUNGSTROCKNEN MIT HILFE VON ELEKTROMAGNETISCHEN FELDERN IM RADIOFREQUENZ- UND MIKROWELLENBEREICH

Zusammenfassung—Es wird ein physikalisches Modell für das Trocknen ungebundener Feuchtigkeit beschrieben, welches die Methode betrifft, elektromagnetische Energie, die bei Radio- oder Mikrowellenfrequenzen erzeugt wird, volumetrisch zu absorbieren. Das Modell bezieht sich auf die Fälle, wo die Temperatur des nassen Feststoffes schnell auf die Verdampfungstemperatur der Flüssigkeit gesteigert werden kann. Verdampfung findet innerhalb der Poren statt und führt zu einer Druckerhöhung, die den Dampf austreibt. Unter Verwendung vereinfachter Beziehungen für den Wärme- und Stoffaustausch werden die Anfangserwärmung, der Druckaufbau und das Trocknen des Feststoffes mit den Eigenschaften des nassen Materials in Beziehung gebracht.

ТЕПЛО- И МАССООБМЕННЫЕ ХАРАКТЕРИСТИКИ ПРОЦЕССА СУШКИ В РЕЖИМЕ КИПЕНИЯ ЖИДКОСТИ ПРИ ВОЗДЕЙСТВИИ РАДИОЧАСТОТНЫХ И МИКРОВОЛНОВЫХ ЭЛЕКТРОМАГНИТНЫХ ПОЛЕЙ

Аннотация — Предложена физическая модель для описания процесса удаления несвязанной стыковой влаги методом, в основу которого положено объемное поглощение электромагнитной энергии, генерируемой при высоких и микроволновых частотах. Модель действительна в тех случаях, когда температура влажного тела может быстро поднята до точки кипения жидкости. Испарение происходит внутри пор и вызывает рост давления, интенсифицирующего удаление образующегося пара. С помощью упрощенных уравнений тепло- и массопереноса нагрев, рост давления и испарение жидкости сопоставлены со свойствами влажного материала.